Synchronous Branch Guide Directional Couplers for Low and High Power Applications*

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Summary—**Branch-guide directional couplers can be built in most types of transmission line. A design procedure is here developed which gives predictable and superior performance over a specified frequency band. A new chart was constructed from which the coupler impedances or admittances can be calculated quickly and with sufficient accuracy for nearly all practical applications.**

A five-branch, 6-db coupler and a thirteen-branch, 0-db coupler were constructed in waveguide. The measured points and computed curves were in excellent agreement. Over the frequency band of 1300 ± 130 Mc, the 0-db coupler had a VSWR of less than 1.07, its insertion loss was better than 0.05 db, and the couplings into the two remaining arms were weaker than 20 db. This coupler can pass at least 5 Mw of peak power in air at atmospheric pressure.

I. INTRODUCTION

DIRECTIONAL couplers at UHF and microwave frequencies take many forms and have many applications. The branch-guide directional coupler, which is the one investigated here, is suitable for construction in almost any kind of transmission line. In waveguide, its mechanical configuration and its electrical performance are similar to those of Riblet short-slot couplers, or of multihole couplers; in coaxial line or strip-line (where only TEM modes exist), the coupler does not have any such close counterparts. Branch-guide couplers have the following useful combination of properties:

1) The coupling between the two lines is through joining branch-lines of finite length, and not through apertures. This gives additional flexibility in design; *e.g.*, special-purpose chokes or filters can be placed in the branches.
2) A branch-guide coupler can be designed either as a periodic structure, or as a band-pass filter; as a band-pass filter, its electrical behavior can be optimized over the pass band.
3) The number of branches can be increased systematically to improve the electrical performance.
4) The coupler is better suited for strong coupling (stronger than 20 db) than weak coupling; 0-db couplers are feasible over large bandwidths.
5) In waveguide, the coupler E-plane cross-section is constant. It can therefore be milled in two blocks and assembled by the "split-block" construction. Other components can also be milled into the same block.
6) The coupler fits into a rectangle; the four output lines are parallel (Fig. 1).

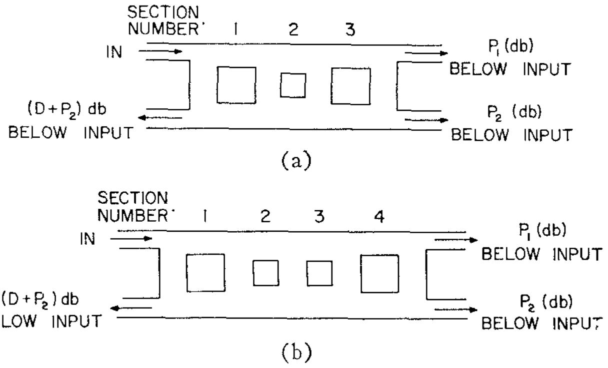

Fig. 1—Branch-guide coupler cross sections—(a) n = odd (3 section), (b) n = even (4 section).

7) Branch-guide couplers are capable of handling high RF powers.
8) They compare favorably with most couplers as regards VSWR, directivity, and constant coupling over large bandwidths.

In this report a design procedure is worked out, based on the quarter-wave transformer as a prototype circuit, which comes close to being a true synthesis procedure: it enables the designer to work out the physical dimensions of his coupler to meet his performance specification with little or no subsequent need for experimental adjustment.

A short review of the literature on branch-guide couplers will indicate the state of the art. The idea of the even and odd mode analysis of couplers having this kind of symmetry (between upper and lower halves in Fig. 1) goes back at least as far as a war-time report by Lippman.[1] This method of analysis has also been explained in several more recent publications.[2,3] The superposition of the even and odd modes, each of which can be solved separately as a loaded-transmission-line

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[1] B. A. Lippmann, "Theory of Directional Couplers," Mass. Inst. Tech., Cambridge, Mass., M.I.T. Rad. Lab. Rept. 860; December 28, 1945.

[2] J. Reed and G. Wheeler, "A method of analysis of symmetrical four-port networks," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-4, pp. 246–252; October, 1956.

[3] L. Young, "Branch guide directional couplers," *Proc. Nat. Electronics Conf.*, vol. 12, pp. 723–732; 1956. (There is a misprint for the 5-branch coupler in Table I. The expression for Y contains a term PKH, which should have been PKH^2.)

(or two-port) problem, then represents the actual directional coupler (or four-port) situation with the generator connected to only a single input. The paper by Reed and Wheeler[2] compares the performance of branch-guide couplers with that of hybrid rings. A later paper by Reed[4] gives general formulas and the results of many calculations on numerous branch-guide couplers, particularly with 0-db coupling. These two papers consider only periodic couplers with uniform main line, that is to say, the branch impedances are all the same except possibly for the end branches, and the main line impedance is constant from input to output. Such couplers have some of the advantages and disadvantages of periodic structures: they are simpler to construct than less regular structures, but they have no clearly defined pass band in which the optimum performance is sought or realized. To realize such band-pass behavior, a filter rather than a periodic structure is needed, and the branch and main line impedances have to be controlled separately. Lomer and Crompton[5] have described an experimental five-branch "binomial" coupler, in which only the branch impedances were adjusted, the main line still being of uniform impedance. Their approach is based on a first-order theory,[6] in which any coupler is considered as a cascaded set of two-branch couplers; extensive empirical changes have to be made experimentally to obtain the desired performance. Young[3] has considered the general case when the branch- and main-line impedances are both allowed to vary. He has given formulas ensuring perfect match and perfect directivity as well as the correct coupling at a single frequency.

This paper supplies an optimization procedure over a given pass band. The performance of many designs was analyzed on a digital computer to check out the theory. Compared to periodic couplers[2,4] only about half as many branches are generally required to give about the same pass-band performance. One 6-db and one 0-db coupler were built in waveguide, with very close agreement between experimental results and predicted performance.

The couplers considered in this paper are herein designated "synchronous" couplers, in the sense that they are derived from a synchronous filter prototype circuit, as will now be explained.

II. THE QUARTER-WAVE TRANSFORMER PROTOTYPE CIRCUIT

A prototype circuit may be defined as a circuit that can be designed to have certain desired electrical characteristics, and that can in some manner be transformed into another circuit having the desired mechanical characteristics while retaining at least approximately the desired electrical characteristics. The prototype circuit is usually in such a form that it can readily be synthesized to meet the electrical performance specifications.

A well-known prototype situation is the transformation of lumped-constant low-pass filters[7,8] into band-pass filters, both lumped-constant and microwave. Another example is the quarter-wave transformer,[9] which can be transformed into half-wave filters and direct-coupled-cavity filters. The general synthesis procedure for quarter-wave transformers is known[10,11] and numerical tables of solutions have been published.[12,13]

The quarter-wave transformer can also be used as a prototype circuit for branch-guide couplers, by appropriately relating the steps of the transformer to the T junctions of the coupler. The notation for the branch-guide coupler impedances or admittances is shown in Fig. 2. For *shunt* stubs an *admittance* representation is used, and for *series* stubs an *impedance* representation. Each T junction becomes a one-eighth-wavelength (or 45°) stub in both the even and the odd mode, open-circuited in the one case and short-circuited in the other, as shown in Fig. 3. Only the shunt case is shown in the lower half of Fig. 3. Then the 45° stub becomes a shunt admittance $\pm jH_i$ at the ith junction counting from either end. The line admittances on either side are K_{i-1} and K_i respectively. The dual of the coupler with shunt junctions is the coupler with series junctions, in which all H and K are impedances. Since we wish to include both cases in one discussion, we shall (following Bode[14]) refer to H and K as *immittances*, meaning admittances when there are shunt junctions, and impedances when there are series junctions. The two nearest reference planes with real reflection coefficient Γ_i are shown in Fig. 3 at distances ϕ_i' to the left and ϕ_i'' to the right of the junction. Without loss in generality, we may suppose

$$K_i > K_{i-1}. \tag{1}$$

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[4] J. Reed, "The multiple branch waveguide coupler," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-6, pp. 398–403; October, 1958.

[5] P. D. Lomer and J. W. Crompton, "A new form of hybrid junction for microwave frequencies," *Proc. IEE* (*London*), *B*, vol. 104, pp. 261–264; May, 1957.

[6] J. W. Crompton, "A contribution to the design of multi-element directional couplers," *Proc. IEE* (*London*), *C*, vol. 104, pp. 398–402; September, 1957.

[7] S. B. Cohn, "Direct-coupled-resonator filters," PROC. IRE, vol. 45, pp. 187–196; February, 1957.

[8] G. L. Matthaei, "Design of wide-band (and narrow-band) band-pass microwave filters on the insertion loss basis," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-8, pp. 580–593; November, 1960.

[9] Leo Young, "The quarter-wave transformer prototype circuit," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-8, pp. 483–489; September, 1960.

[10] H. J. Riblet, "General synthesis of quarter-wave impedance transformers," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-5, pp. 36–43; January, 1957.

[11] L. Young, "Synthesis of multiple antireflection films over a prescribed frequency band," *J. Opt. Soc. Am.*, vol. 51, pp. 967–974; September, 1961.

[12] L. Young, "Tables for cascaded homogeneous quarter-wave transformers," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-7, pp. 233–237; April, 1959, and vol. MTT-8, pp. 243–244; March, 1960.

[13] L. Young, "Stepped impedance transformers and filter prototypes," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-10, pp. 339–359; September, 1962.

[14] H. W. Bode, "Network Analysis and Feedback Amplifier Design," D. Van Nostrand Co., New York, N. Y.; September, 1945.

Then it can be shown (for instance, by referring to a Smith chart) that

$$0 \leq \phi_i' \leq \phi_i'' \leq 90°, \quad \text{when} \quad H_i > 0 \tag{2a}$$

and

$$0 \leq \phi_i'' \leq \phi_i' \leq 90°, \quad \text{when} \quad H_i < 0. \tag{2b}$$

The values of ϕ_i' and ϕ_i'' are given in terms of H_i, K_{i-1} and K_i by

$$\left.\begin{aligned} \phi_i' &= \frac{1}{2}\arc\tan\left(\frac{2H_iK_{i-1}}{H_i^2+K_i^2-K_{i-1}^2}\right) \\ \phi_i'' &= \frac{1}{2}\arc\tan\left(\frac{2H_iK_i}{H_i^2+K_{i-1}^2-K_i^2}\right) \end{aligned}\right\}. \tag{3}$$

Furthermore, when H_i is positive, and when the right-hand (high-K) side is matched, then the normalized immittance looking into the reference plane A on the left-hand (low-K) side is real and greater than unity; when the left-hand (low-K) side is matched, then the normalized immittance looking into the reference plane B on the right-hand (high-K) side is likewise real and greater than unity. However, when H_i is negative the normalized immittances seen in the two nearest reference planes A and B are real but less than unity. (These results can all be proved by consulting a Smith chart.)

The junction VSWR[9,13] V_i is given by

$$V_i = \frac{[(K_i+K_{i-1})^2+H_i^2]^{1/2}+[(K_i-K_{i-1})^2+H_i^2]^{1/2}}{[(K_i+K_{i-1})^2+H_i^2]^{1/2}-[(K_i-K_{i-1})^2-H_i^2]^{1/2}}. \tag{4}$$

A sequence of junctions like the one shown in Fig. 3 (shunt case) is shown in Fig. 4. It represents a portion of one-half the branch-guide coupler in either the even or odd mode (either all H_i positive or all H_i negative). If the branch-guide coupler (*i.e.*, the circuit of Fig. 4) is to be based on the quarter-wave transformer, reference planes on opposite sides of adjacent junctions must touch[15] as shown in Fig. 4, which is the synchronous-tuning condition[13] for filters. Branch-guide couplers designed in this manner will therefore be called *synchronous couplers.*

In addition, the junction VSWRs, V_i, of the coupler must be set equal to the V_i of the selected prototype transformer, which gives a condition connecting K_i, K_{i-1} and H_i. (Only their ratios, and not the impedance level, are significant, so that only two and not three quantities have to be solved for.) The other condition derives from the reference-plane positions. Since the coupler is symmetrical about the center, the position of the reference plane associated with the center branch or pair of branches depends on whether the number of sections, n, is odd or even (Fig. 1). (The number of branches is one more than the number of sections, that is, $n+1$.)

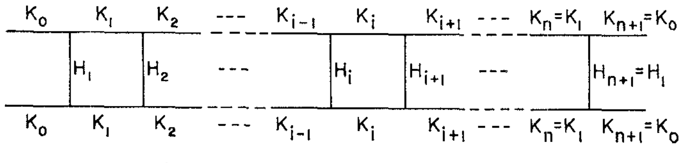

Fig. 2—Branch-guide coupler notation.

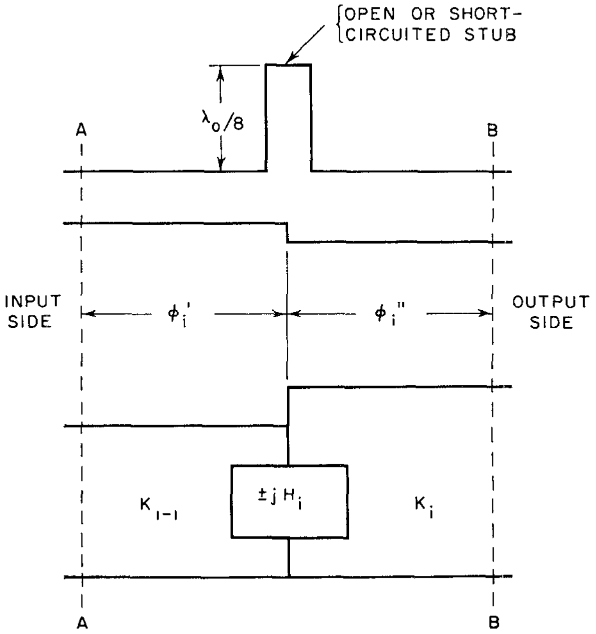

Fig. 3—Equivalent circuit of *i*th *T*-junction for even or odd mode at center frequency.

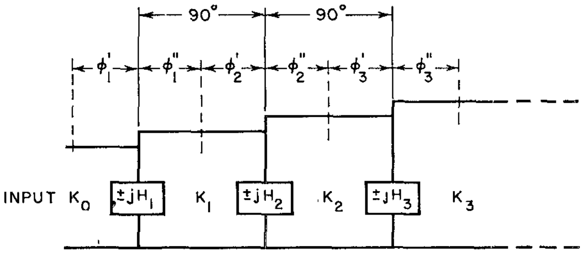

Fig. 4—Spacings between branches for synchronous couplers.

For n = odd [*Fig. 1(a)*]—Suppose for instance that $n=3$. There are thus four branches. The center reference plane at band center is by symmetry 45° from its junction, which in this case is the second junction from the end. Once a three-section quarter-wave transformer prototype has been selected, V_2 of this prototype transformer can be calculated. For $n=3$, or any odd n, $\phi_2''=45°$, and from (3) and (4) the parameters of this junction, K_1/K_2, H_2/K_2 and ϕ_2', can now be deduced. This in turn yields $\phi_1''=90°-\phi_2'$, which, when V_1 of the prototype transformer is known, yields the required parameters of the first junction, K_0/K_1 and H_1/K_1, from (3) and (4).

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[15] This is possible for both the even and the odd modes simultaneously when the branch lengths and the branch spacings are all one-quarter wavelength. It is probably not possible in any other case.

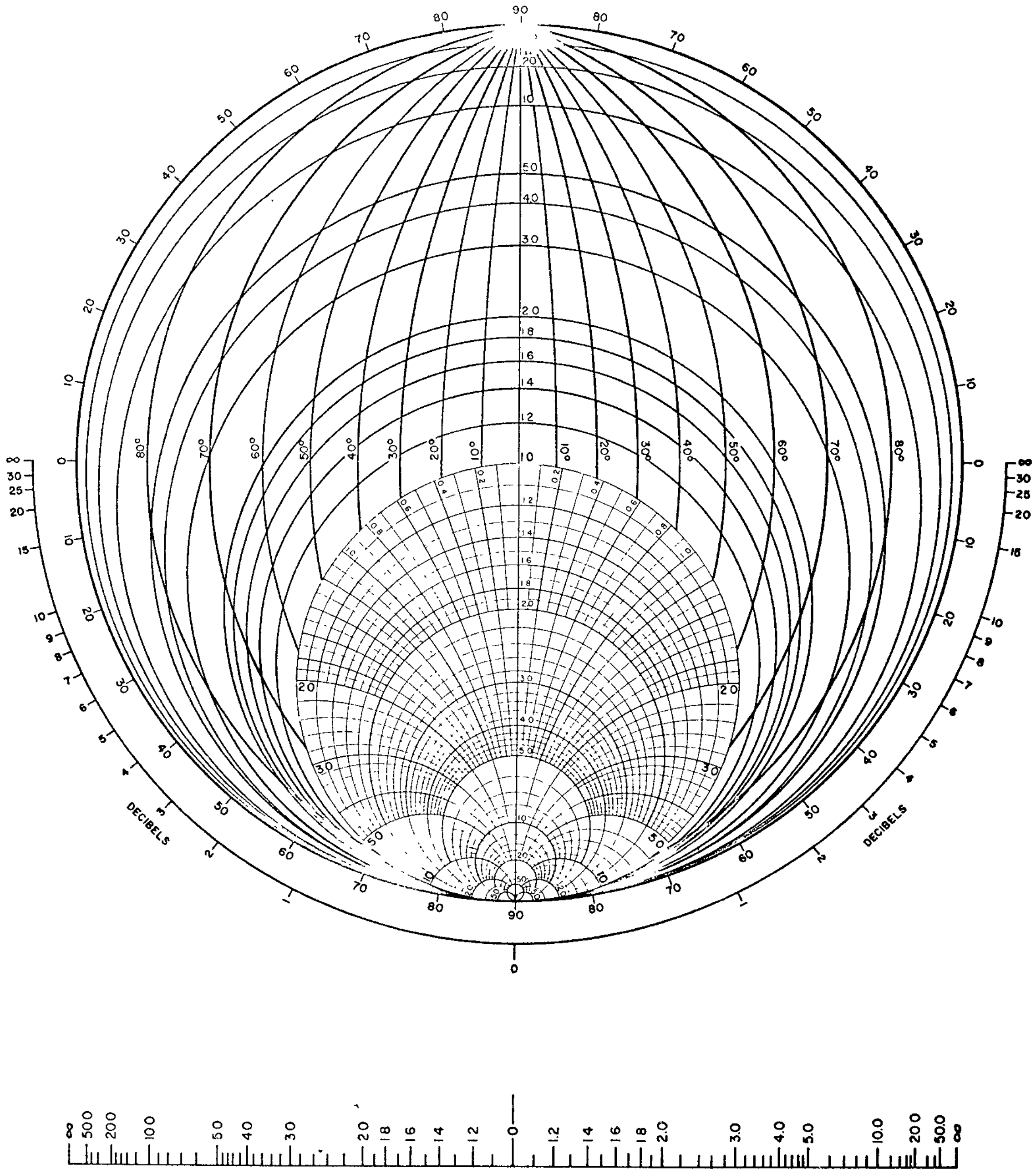

Fig. 5—Design chart for branch-guide coupler—complete chart.

For n = even [*Fig. 1(b)*]—Suppose that for instance $n=4$. There are thus five branches. The center branch, of immittance H_3, has the same immittance K_2 on either side of it, since n is even. The VSWR of the immittance $(1+jH_3/K_2)$ is set equal to the junction VSWR, V_3, of the middle step (the third step from either end) of the appropriate four-section quarter-wave transformer prototype circuit. This determines H_3/K_2 and also $\phi_3'=\phi_3''$ of the middle junction. From the next junction VSWR, V_2, of the prototype transformer and the equation $\phi_2''=90°-\phi_3'$, the parameters of the second T junction are obtained from (3) and (4), yielding K_1/K_2, H_2/K_2 and ϕ_2', and so on down the line.

Since this procedure is numerically tedious, a graphical solution was devised, which depends on a sort of Smith chart[16] crossed with a Carter chart.[17] The graphical solution is described in Section III.

III. Design by Charts

The three charts shown in Figs. 5–7 are, respectively, the full chart and two charts with an expanded center. (Fig. 7 is more expanded than Fig. 6.) The lower circle in Fig. 5 contains the portion of a Smith chart inside the

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[16] P. H. Smith, "Transmission line calculator," *Electronics*, vol. 12, pp. 29–31; January, 1939; and "An improved transmission line calculator," *Electronics*, vol. 17, pp. 130–133 and 318–325; January, 1944.

[17] P. S. Carter, "Charts for transmission-line measurements and computations," *RCA Rev.*, vol. 3, pp. 355–368; January, 1939.

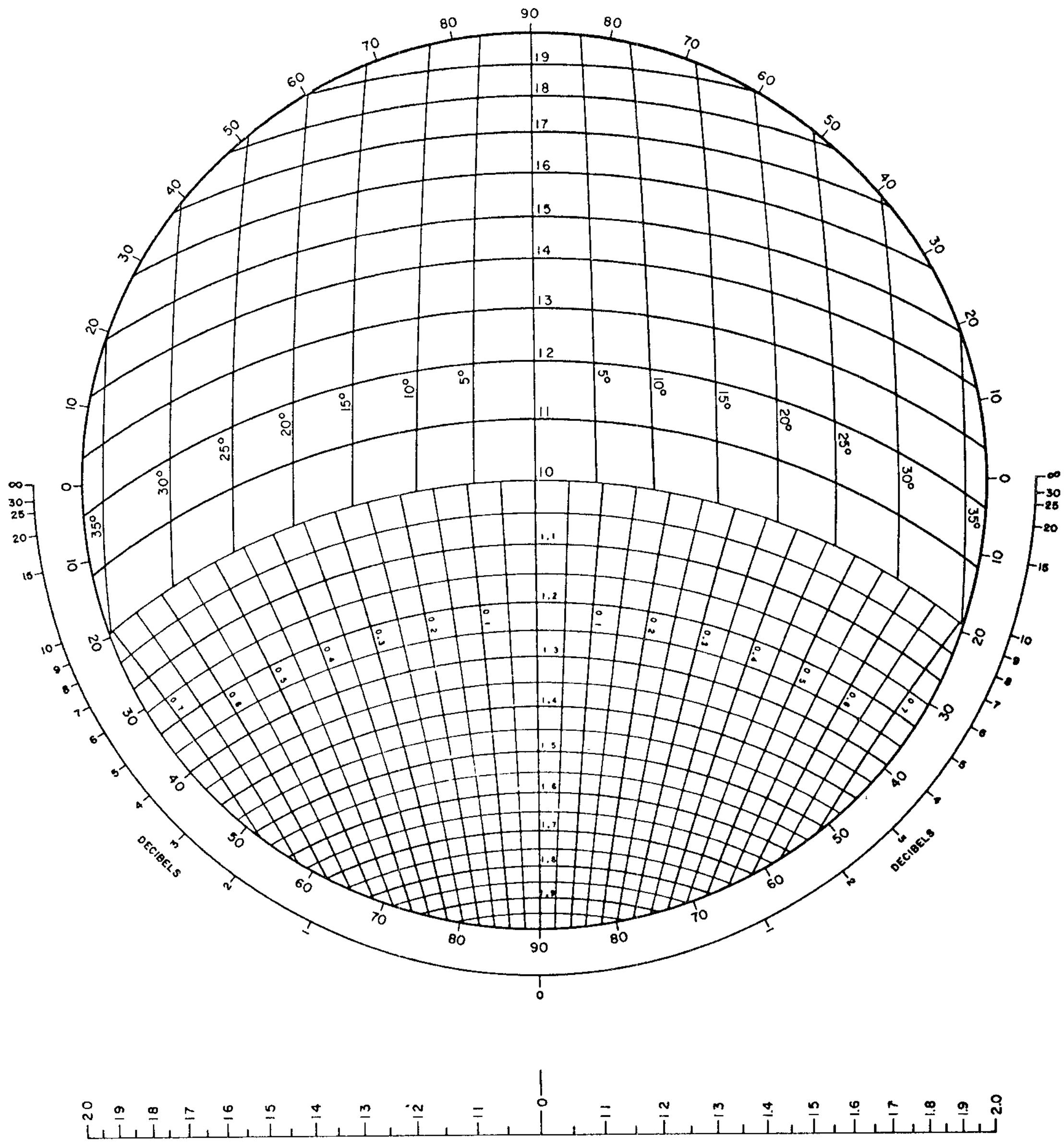

Fig. 6—Design chart for branch-guide coupler—center expanded to VSWR = 2.0.

unit conductance (or resistance) circle. There are two families of circles outside. One is the family of constant-conductance[18] circles, with the reciprocal values marked; thus inside the Smith chart circle the conductances have been selected to have values 1.2, 1.4, 1.6, etc., while those outside are marked likewise but have conductance 1/1.2, 1/1.4, 1/1.6, etc. (instead of the usual circles of conductance 0.8, 0.6, etc.). The other family of circles represents contours of arg Y = constant where Y stands for admittance. If we write $Y = G + jB$, then the number of degrees represents the quantity arc tan (B/G). The use of this chart will now be explained by means of two examples, and the theory behind it will become apparent as the explanation proceeds.

Example III-1—Case of n = odd

Design a branch-guide coupler based on a quarter-wave transformer of $n = 3$ sections, output-to-input impedance ratio $R = 6$, and transformer fractional bandwidth[19] $w_q = 0.6$. (The selection of the prototype is dealt with later.)

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[18] To simplify the explanation, only conductance and susceptance will be used (corresponding to shunt branches); the statements are however equally true on the dual impedance basis (corresponding to series branches).

[19] The transformer fractional bandwidth is defined as usual by

$$w_q = 2\frac{\lambda_{g1}' - \lambda_{g2}'}{\lambda_{g1}' + \lambda_{g2}'}$$

where λ_{g1}' and λ_{g2}' are the longest and shortest guide wavelengths in the quarter-wave-transformer pass band.[12] Its center-frequency is determined from the guide wavelength $\lambda_g{}'_0$ at center frequency, given by

$$\lambda_{g0}' = \frac{2\lambda_{g1}'\lambda_{g2}'}{\lambda_{g1}' + \lambda_{g2}'}.$$

The response is symmetrical about this point when plotted against λ_{g0}'/λ_g'.

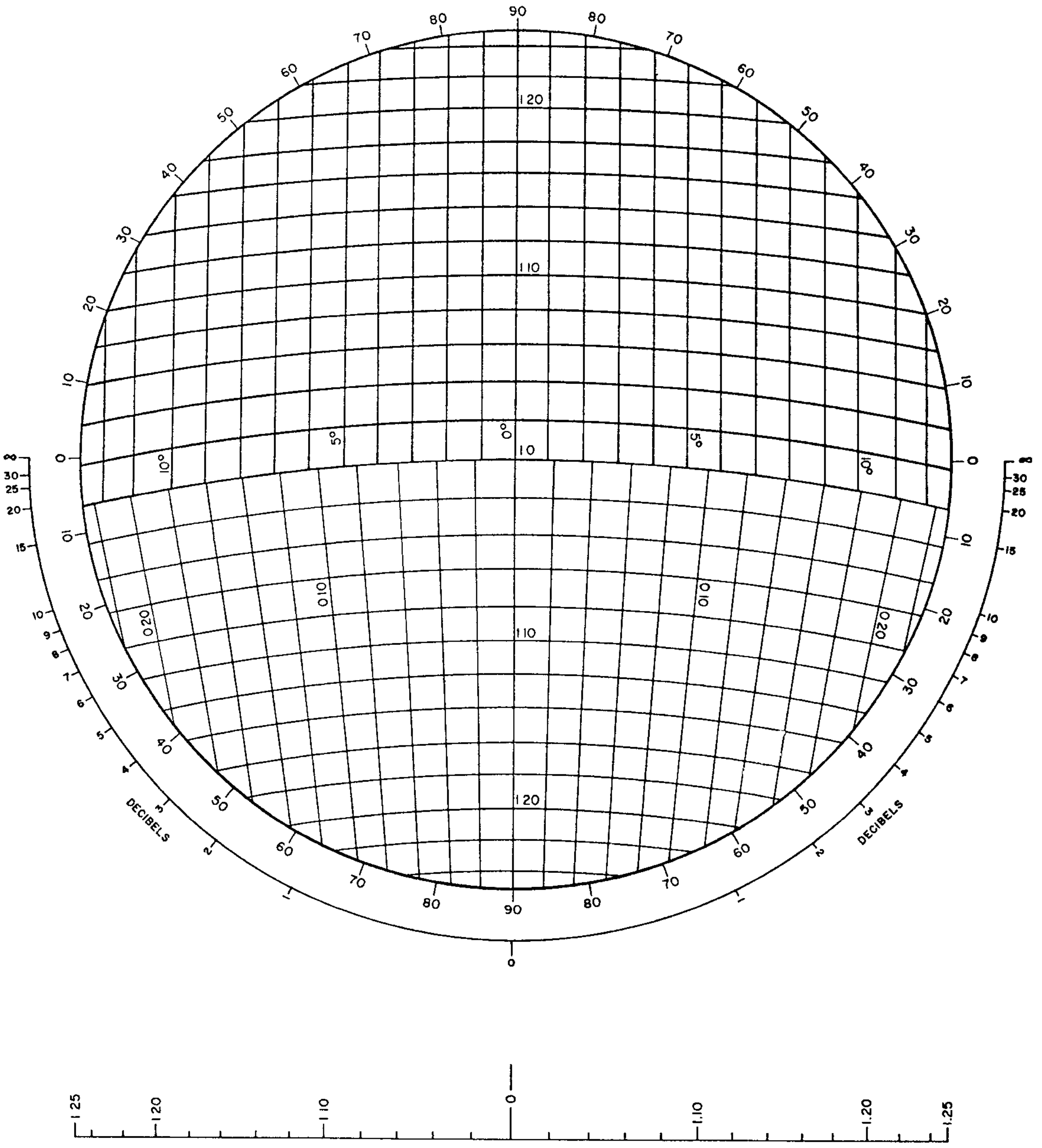

Fig. 7—Design chart for branch-guide coupler—center expanded to VSWR = 1.25.

From the tables,[12] the junction VSWRs of the prototype transformer are $V_1 = 1.311$ and $V_2 = 2.4495/1.311 = 1.869$. Half the coupler for either the even or the odd mode is shown in Fig. 8.

The second junction from the end is 45° from the reference plane at the center of the coupler, where the reflection coefficient of this junction (Γ_2) is real. Therefore in the plane of the junction, Γ_2 is pure imaginary (since it is 45° from Γ_2 = real), and this Γ_2 is therefore on the horizontal diameter in Fig. 9. Its position is determined by $V_2 = 1.869$, and it is located at the point marked "START." It corresponds to the normalized admittance $(K_1 \pm jH_2)/K_2$. Next we wish to find the admittance $A_2 + jB_2 = (K_2 \pm jH_2)/K_1$ which corresponds to the junction admittance seen from the other side in Fig. 8. (Admittance is again used to simplify the explanation. For a coupler with series junctions, replace admittance, conductance and susceptance, by impedance, resistance and reactance, respectively.) It is obtained as indicated in Fig. 9 by first following the arrowed line along an "arg Y = constant" contour, down to the unit conductance circle, and thence following a constant susceptance contour as far as the circle $Y = K_2/K_1$. By stopping on this circle we ensure inverting the conductance component from K_1/K_2 at the start to $A_2 = K_2/K_1$. That the susceptance component comes out as it should can be deduced from the fact that the lower portion of the arrowed path keeps the susceptance constant, by definition, while the upper portion follows an "arg Y = constant" contour, and therefore the real and imag-

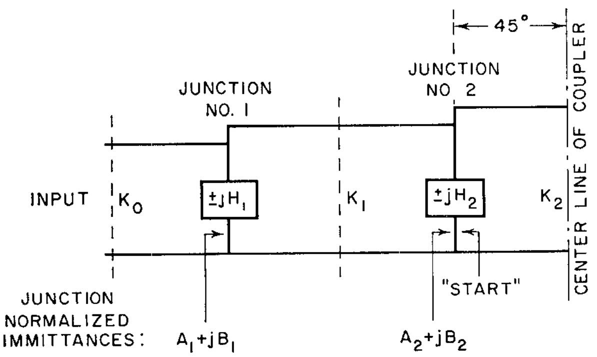

Fig. 8—Even- or odd-mode equivalent circuit for design of four-branch ($n=3$) coupler used in Example III-1.

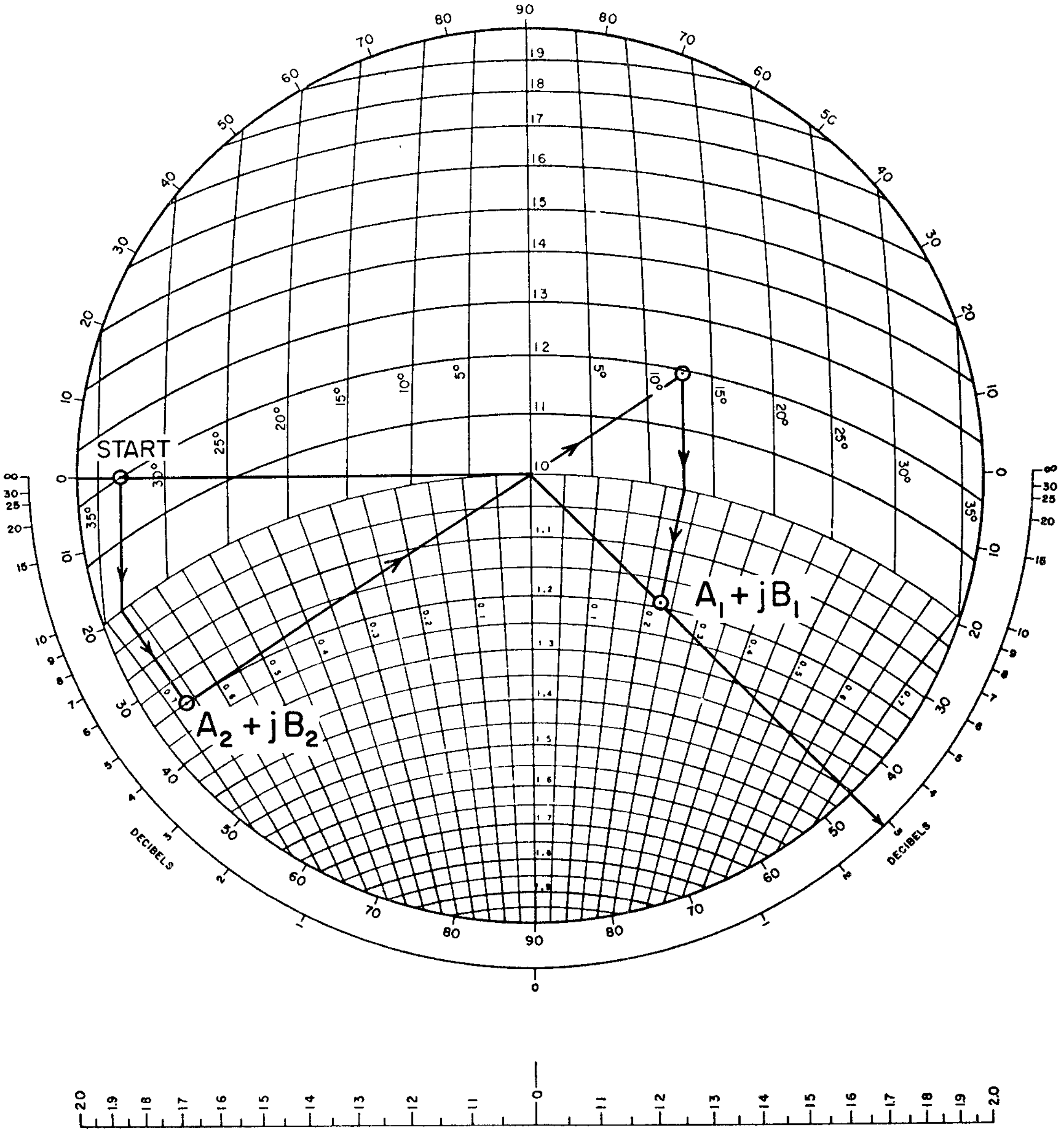

Fig. 9—Solution by chart of four-branch ($n=3$) coupler used in Example III-1.

inary parts of Y are multiplied up or down together. Now the real part changes from K_1/K_2 to unity (following the arrow), and therefore the imaginary part is also multiplied by K_2/K_1 and this is just what is needed to turn jH_2/K_2 into jH_2/K_1. Incidentally, the two points marked "START" and A_2+jB_2 are necessarily at the same radius, since the normalized junction reflection coefficient, or the junction VSWR, of a lossless junction is independent of which port is taken as the input side.

To move on to junction 1, the restriction $\phi_1''=90°-\phi_2'$ determines the direction of the radius in Fig. 9. It points into the upper left-hand quadrant, and images the radius to A_2+jB_2 in the horizontal line. For construction purposes it is easier to ignore differences between left and right of the vertical center line, and simply to continue the radius from A_2+jB_2 along a straight line following the arrow into the upper *right*-hand quadrant in Fig. 9. The distance out to the circle there corresponds to a VSWR of $V_1=1.311$. On a full Smith chart this point would give the admittance in the plane of the first junction seen from the line K_1, *i.e.*, the admittance $(K_0 \pm jH_1)/K_1$. By the same construction as before we now turn this admittance into $A_1+jB_1=(K_1 \pm jH_1)/K_0$, which is the admittance of the first junction seen from the line K_0. All the coupler admittances are now found as follows, normalized to $K_0=1.0$:

$$K_1 = A_1 = 1.189 \qquad H_1 = B_1 = 0.228$$
$$K_2 = A_2K_1 = 1.429 \qquad H_2 = B_2K_1 = 0.7925 \tag{5}$$

where B_1 and B_2 are simply taken as positive, since differences between the left and right half are being ignored. In general, the solution when the number of sections, n, is odd (when the number of branches, $n+1$, is even) is obtained from

$$\left.\begin{aligned} K_0 &= 1 \\ K_1 &= A_1 \\ K_2 &= A_1A_2 \\ K_3 &= A_1A_2A_3 \\ K_4 &= A_1A_2A_3A_4 \\ &\text{etc., and} \\ H_1 &= B_1 \\ H_2 &= A_1B_2 \\ H_3 &= A_1A_2B_3 \\ H_4 &= A_1A_2A_3B_4 \end{aligned}\right\} \tag{6}$$

etc.

If the coupler consists of series instead of shunt junctions, then K and H are impedances instead of admittances.

The coupling coefficient at center frequency for a matched coupler can also be read off from Fig. 9, by extending the radius through (A_1+jB_1) to the outermost scale. In this case it is 2.9 db. The coupling when the input is matched is given by the difference in phase shifts suffered by the even and odd modes. Now the phase shift between reference planes (with real Γ) is a multiple of 90°, and is the same for both modes. It is therefore necessary to calculate the separation between the even- and odd-mode reference planes at either end of the coupler. These separations are the same at the two ends of the coupler, by symmetry, and are in equal and opposite directions for the two modes from the position corresponding to no coupling. The coupling C (db) at center frequency (where the coupler is perfectly matched since n is odd) is finally found to be equal to the square of the sine of the angle between the radius to A_1+jB_1 and the horizontal axis. Expressed in decibels, it is marked off on the lower outer scale in Fig. 9. It is seen that for 0-db coupling (complete cross-over) to be possible, all of the points $A_i \pm jB_i$ would have to lie on the vertical axis. It will in fact be seen later [(11a)] that 0-db coupling with a single design becomes possible only in the limit of R tending to infinity. A 0-db coupler has therefore to be designed as two or more couplers in cascade, *e.g.* two 3-db couplers, or three 6-db couplers, etc.

Example III-2—Case of n = even

Design a branch-guide coupler based upon a quarter-wave transformer of $n=4$ sections, output-to-input impedance ratio $R=6$, and bandwidth $w_q=1.00$.

From the tables[12] the junction VSWRs of the prototype transformer are $V_1=1.247$, $V_2=1.518$, $V_3=1.672$. Half the coupler for either the even or the odd mode is shown in Fig. 10. The third junction from the end is the middle junction, and the lines on either side of it have the same admittances, K_2. The junction admittance seen from either side is $1 \pm jH_3/K_2$; its real part is thus unity, and it must correspond to a VSWR of $V_3=1.672$. This is the point marked "START" in Fig. 11, situated on the unit conductance circle. Apart from this different beginning, all subsequent steps are as in Example III-1, and it is found that:

$$K_0 = 1.0 \qquad H_1 = B_1 = 0.180$$
$$K_1 = A_1 = 1.155 \qquad H_2 = B_2K_1 = 0.481$$
$$K_2 = A_2K_1 = 1.383 \qquad H_3 = B_3K_2 = 0.719. \tag{7}$$

The general solution is still given by (6). The coupling C (db) at center frequency (where this coupler has a very low VSWR) is read off as before and is again 2.9 db. It will be seen later [(11a)] that the coupling is only a function of R, when the reflection loss is negligible, and since $R=6$ in both examples, this result was to be expected. An expression for the actual coupling $P_{2,0}$ (db) when there is appreciable reflection loss will also be given later [(11b)].

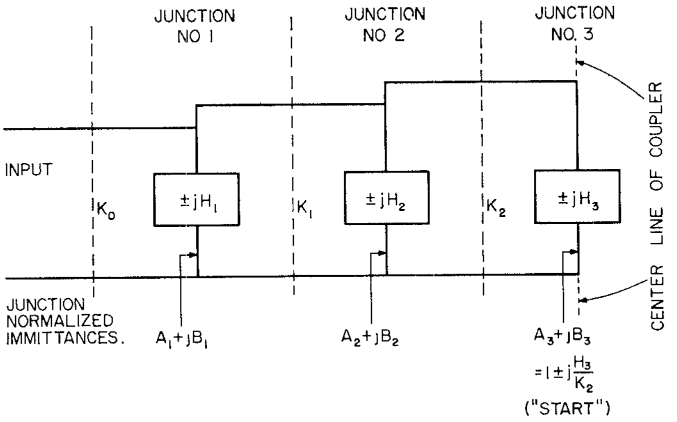

Fig. 10—Even- or odd-mode equivalent circuit for design of five-branch ($n=4$) coupler used in Example III-2.

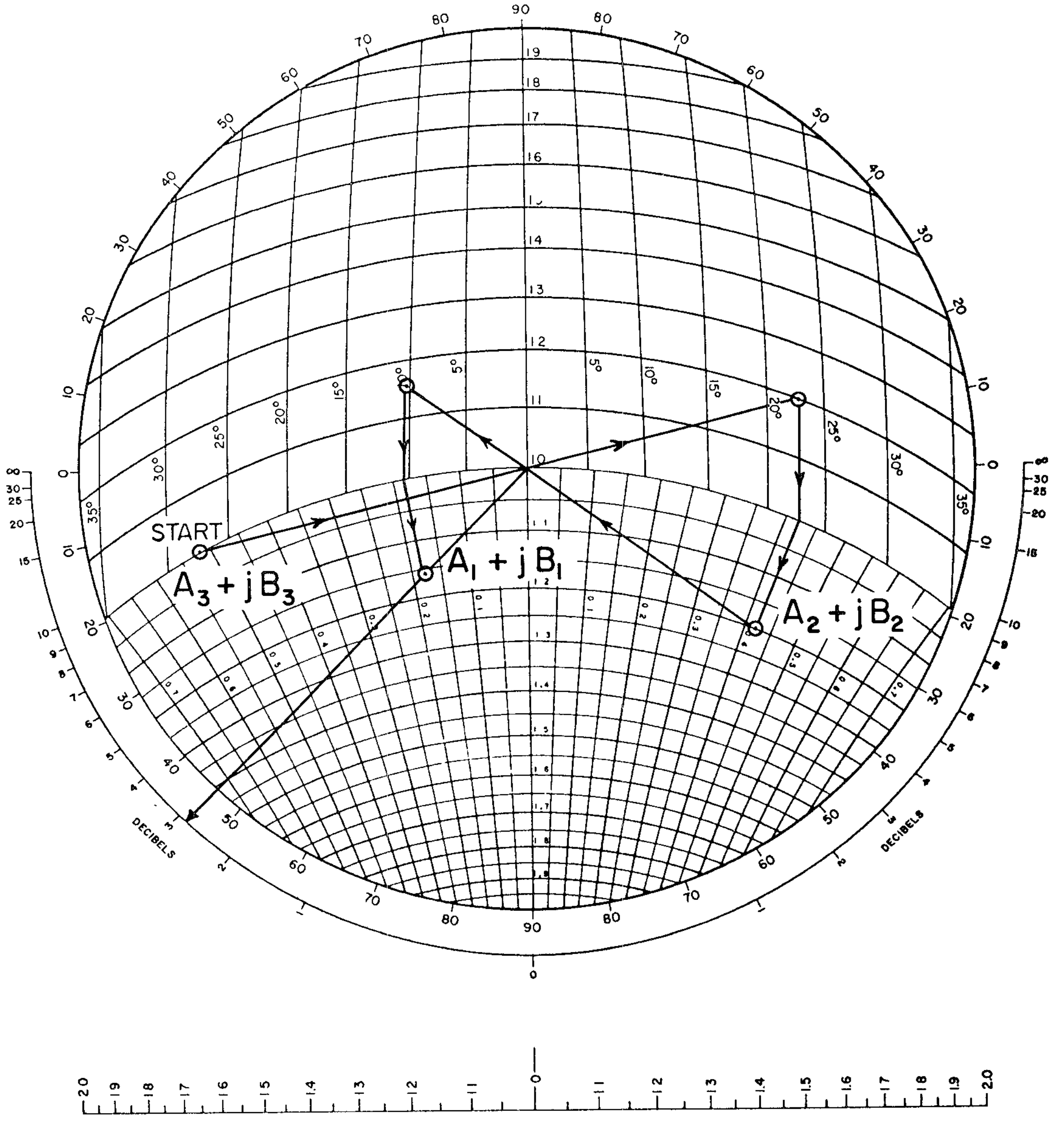

Fig. 11—Solution by chart of five-branch ($n=4$) coupler used in Example III-2.

IV. Choice of Quarter-Wave Transformer Prototype

The even- or odd-mode half of the branch-guide coupler has been modeled on the quarter-wave transformer at center-frequency. If the prototype transformer has a wide-band Chebyshev response, then the branch-guide coupler may similarly be expected to have low VSWR and high directivity over a wide band of frequencies; if the prototype transformer is narrow-band or maximally flat, then the branch-guide coupler VSWR and directivity response may be expected to be narrow-band or approximately maximally flat. The coupling from one line over to the other (P_2 on Fig. 1) is generally found to increase slowly as the frequency moves away from band center (either up or down), and is less obviously dependent on the prototype characteristics. One would like to have some criteria leading from the specification for the coupler to the selection of the prototype transformer that will transform it into a coupler meeting the specification.

Let V and Γ be the VSWR and associated reflection coefficient of the branch-guide coupler at any frequency; let V' and Γ' be the VSWR and associated reflection coefficient of the prototype quarter-wave transformer at the corresponding frequency. An examination of the phase relationships between reflected and transmitted waves[17] shows that, at center frequency,

$$\left.\begin{aligned} \Gamma &= \frac{1}{(1-\Gamma^2)^{1/2}} \cdot \frac{\Gamma'}{\text{antilog}\,(P_2/20)} \\ &= \frac{V+1}{2V^{1/2}}\,\frac{\Gamma'}{\text{antilog}\,(P_2/20)} \end{aligned}\right\} \qquad (8a)$$

and the directivity[20] D in decibels is given by

$$\left.\begin{aligned} D &= -20\log_{10}\left[\frac{1}{(1-\Gamma^2)^{1/2}}\,\frac{\Gamma'}{\text{antilog}\,(P_1/20)}\right] - P_2 \text{ db} \\ &= -20\log_{10}\left[\frac{V+1}{2V^{1/2}}\,\frac{\Gamma'}{\text{antilog}\,(P_1/20)}\right] - P_2 \text{ db} \end{aligned}\right\}. \qquad (9a)$$

Equations (8a) and (9a) also hold approximately *near* center frequency.

We shall now define a *bandwidth contraction factor*, β, as follows: If the prototype transformer fractional bandwidth is w_q, over which its VSWR does not exceed $V'_{\max}$ (the associated reflection coefficient being $\Gamma'_{\max}$), then the branch-guide coupler fractional bandwidth is βw_q, over which its VSWR does not exceed $V_{\max}$ (the associated reflection coefficient being $\Gamma_{\max}$), and its directivity is better than $D_{\min}$ decibels.

Equations (8a) and (9a) hold approximately when $V_{\max}$, $V'_{\max}$, $\Gamma_{\max}$, $\Gamma'_{\max}$ and $D_{\min}$ are substituted for V, V', Γ, Γ', and D respectively. In most cases, $V'_{\max}$ will be close to unity (*i.e.*, $\Gamma'_{\max}$ will be small compared to unity); neglecting Γ^2 compared to unity, (8a) and (9a) reduce to

$$\Gamma_{\max} \approx \frac{\Gamma_{\max}'}{\text{antilog}\,(P_2/20)} \qquad (8b)$$

$$D_{\min} \approx -20\log_{10}\left[\frac{\Gamma_{\max}'}{\text{antilog}\,(P_1/20)}\right] - P_2. \qquad (9b)$$

Making the additional convenient approximation when Γ is small, and $\Gamma \approx (V-1)/2$,

$$V_{\max} \approx 1 + \frac{V_{\max}' - 1}{\text{antilog}\,(P_2/20)} \qquad (8c)$$

$$D_{\min} \approx -20\log_{10}\left[\frac{V_{\max}' - 1}{2\,\text{antilog}\,(P_1/20)}\right] - P_2. \qquad (9c)$$

One can combine (8a) and (9a) to obtain, at center frequency (this becomes an approximation *near* center frequency),

$$\frac{10^{-(D+P_2)/20}}{\Gamma} = \frac{\text{antilog}\,(P_2/20)}{\text{antilog}\,(P_1/20)}. \qquad (10)$$

[*Note:* In (8), (9), and (10) P_1 and P_2 are positive quantities and the antilogs are quantities greater than unity. For instance, for a 6-db coupler, $P_2 = 6$ db, and antilog $(P_2/20) = 2$. If the coupler is also matched, then $P_1 = 1.25$ db, and antilog $(P_1/20) = 1.154$.]

The ratios on either side of (10) are the same quantity as the ratio $E_4/E_3 = X/Y$ in Young[3] and can be calculated from the formulas given there.

Eqs. (8)–(10) apply to synchronous couplers, but not necessarily to others. For example, they do not generally apply to periodic couplers.[4]

When the coupler is matched at center frequency, then the coupling P_2 at center frequency will be denoted by C (db). It can be related to R by the formula

$$C = 20\log_{10}\left[\frac{R+1}{R-1}\right] \text{db}. \qquad (11a)$$

in all the numerical solutions attempted. A general proof for this formula has not yet been found. The coupling C does not depend on the number of branches or the bandwidth. The relation between C and R is graphed in Fig. 12.

When the coupler is not matched at center frequency, and its prototype transformer has a VSWR V_0' at center frequency, then the center frequency coupling $P_{2,0}$ is given by

$$P_{2,0} = C - 10\log_{10}\left[\frac{(V_0'+1)^2}{4V_0'}\right] \text{db} \qquad (11b)$$

in all the numerical solutions attempted.

The branch-guide coupler characteristics (like those of the prototype transformer) will be symmetrical about the center frequency when plotted against λ_{g0}/λ_g, where λ_g is any guide wavelength, and λ_{g0} is the particular guide wavelength at band-center, defined as follows:

---

[20] *i.e.*, the output from the port immediately below the input port in Fig. 1 is $(D+P_2)$ decibels below the input power.

Let λ_{g1} and λ_{g2} be the longest and shortest guide wavelengths in the pass band; then λ_{g0} is defined by

$$\lambda_{g0} = \frac{2\lambda_{g1}\lambda_{g2}}{\lambda_{g1} + \lambda_{g2}}. \tag{12a}$$

The fractional bandwidth of the branch-guide coupler will be denoted by w_b and is defined (analogously to the quarter-wave transformer fractional bandwidth, w_q) by

$$w_b = 2\left(\frac{\lambda_{g1} - \lambda_{g2}}{\lambda_{g1} + \lambda_{g2}}\right). \tag{12b}$$

It has been found that the coupler fractional bandwidth w_b is always less than the prototype transformer fractional bandwidth w_q. Their ratio is denoted by

$$\beta = \frac{w_b}{w_q} \tag{13}$$

which is the *bandwidth contraction factor* already referred to.

Before we can select the appropriate quarter-wave transformer prototype from which to derive our branch-guide coupler, we have to know the bandwidth contraction factor, β. An examination of a large number of cases from maximally flat prototypes to prototypes having bandwidths of at least 80 per cent, showed that the bandwidth contraction factor β did not change appreciably with bandwidth, but did change with the number of sections n (number of branches $= n+1$), and the impedance-ratio parameter R. Because predicting β from the T-junction properties seemed too formidable an undertaking, a large number of branch-guide couplers were analyzed and their bandwidths compared to those of their prototype transformers. From this comparison the graph of Fig. 13 was prepared. The bandwidth contraction factor β was found to lie between 0.5 and 0.7 in most cases; it was nearer the upper value of 0.7 for weaker couplings (smaller R) and fewer sections (lower n). An example of the use of Fig. 13 in the selection of a prototype will now be given.

Example IV-1—Design of a 3-db Coupler

Find the prototype transformer for a 3-db branch-guide coupler which is to have an input VSWR below 1.10 and directivity in excess of 20 db over a 24 per cent fractional bandwidth.

From Fig. 12 or Table I (page 474), when $C = 3$ db, then $R = 5.84$. Try a two-branch coupler first, corresponding to a single-section quarter-wave transformer ($n = 1$). From Fig. 13, $\beta = 0.64$ for $n = 1$, so that the prototype fractional bandwidth must be $w_b/\beta = 24/0.64$ per cent, or nearly 40 per cent by (13). The maximum VSWR of a single-section quarter-wave transformer[12] of $R = 6$ and $w_q = 0.40$, is 1.860. It follows from (8b) that $V_{\max}$ of the coupler would then be considerably greater than the 1.10 specified.

Try a three-branch coupler next, corresponding to a two-section quarter-wave transformer ($n = 2$). From Fig. 13, $\beta = 0.62$ for $n = 2$, so that the prototype fractional bandwidth must be 24/0.62 per cent, or almost 40 per cent. Now the maximum VSWR of a two-section quarter-wave transformer[12] of $R = 6$, $w_q = 0.40$, is 1.11, which for a 3-db coupler by (8c) yields $V_{\max} \approx 1 + 0.11/1.414 = 1.08$ which is below the 1.10 specified. The directivity from (9b) or (9c) will be better than $-20 \log_{10} (0.04) - P_2 = 25$ db which exceeds the 20 db specified.

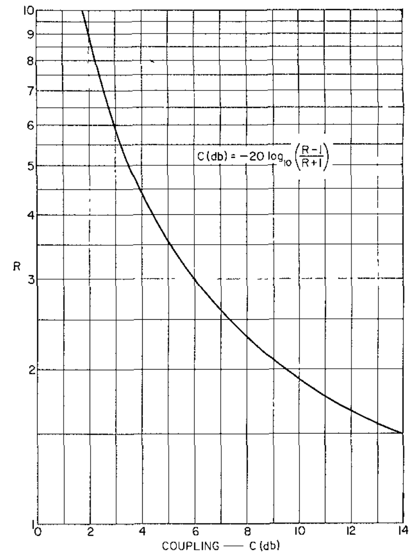

Fig. 12—Plot of center frequency coupling C (db) vs impedance ratio parameter R for a matched coupler.

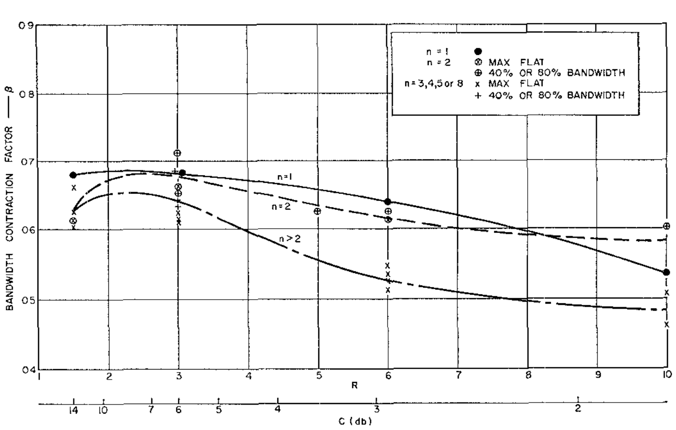

Fig. 13—Best estimates for bandwidth contraction factor β based on 27 individual solutions.

Thus the prototype quarter-wave transformer will in this case have two sections ($n = 2$), $R = 5.84$, and bandwidth $w_q = 0.40$. Its junction VSWRs can be found from tables,[12] and then converted to the branch-guide coupler parameters by means of the charts (see Examples III-1 and III-2 of Section III).

Nothing has yet been said about the variation of the couplings P_1 and P_2 with frequency. This is analogous to prescribing the amplitude characteristic of a filter and then asking about its phase (or time-delay) characteristic. The amplitude and phase characteristics are

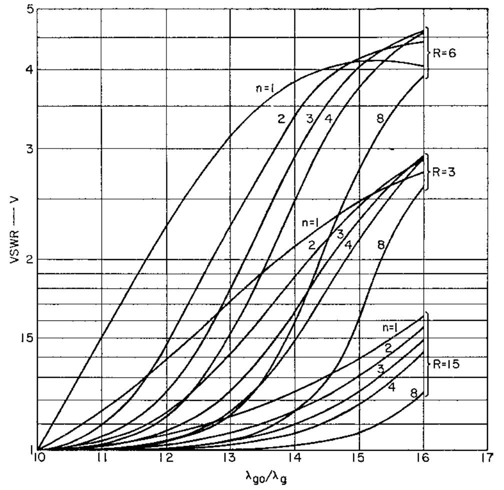

Fig. 14—VSWR characteristics of some maximally flat couplers. (Immitances of these couplers are given in Tables I and II.)

related and cannot be prescribed independently of each other. In the case of the branch-guide coupler the coupling characteristics are determined by the *difference* between the even- and odd-mode phase characteristics. To obtain a systematic picture of the frequency variation of the VSWR V, the directivity D (db) and the two couplings (the in-line coupling P_1 and the cross-over coupling P_2, both in db), these were plotted in Figs. 14–16 for couplers based on *maximally flat* quarter-wave transformer prototypes for $n=1$, 2, 4, and 8, and for $R=1.5$, 3, and 6, corresponding to couplings (C) of approximately 14, 6 and 3 db.

The graphs are plotted against the quantity (λ_{g0}/λ_g), where λ_g is the guide wavelength, and λ_{g0} is its value at band-center [(12a)]. The response is symmetrical about $(\lambda_{g0}/\lambda_g)=1$, so that Figs. 14–16 actually cover the range from 0.4 to 1.6, although only the portion from 1.0 to 1.6 is shown. For nondispersive (TEM mode) lines, the guide wavelength λ_g reduces to the free space wavelength λ, and then λ_{g0}/λ_g reduces to $\lambda_0/\lambda=f/f_0$, where f is the frequency, and f_0 is its value at band-center (also called the center frequency).

It can be seen from Fig. 16 that the coupling P_2 generally becomes stronger (P_2 measured in decibels decreases) on either side of center frequency (the curves are symmetrical about $\lambda_{g0}/\lambda_g=1$), and correspondingly P_1 becomes weaker (P_1 measured in decibels increases).

Now return to Example IV-1. If we may use Fig. 16 as a guide, then over the 24 per cent band specified this three-branch 3-db coupler would be expected to change each of its couplings, P_1 and P_2, by a little under 0.3 db. Thus, if the coupler were designed to have 3-db coupling at center frequency (corresponding to $R=5.84$ picked before), then P_2 would go to 2.7 db and P_1 to 3.3 db at the 24 per cent band edges. If the specification asked for both P_1 and P_2 to be maintained to within ± 0.15 db of 3 db over the 24 per cent band, or generally to optimize the balance over the band as a whole, then the coupler would be designed with $P_{2,0}=3.15$ db, corresponding to $R=5.7$, by Fig. 12 or (11a). This coupler was designed, and has the following parameters: $K_0=1$, $K_1=1.2902$, $H_1=0.4363$, $H_2=1.0844$.

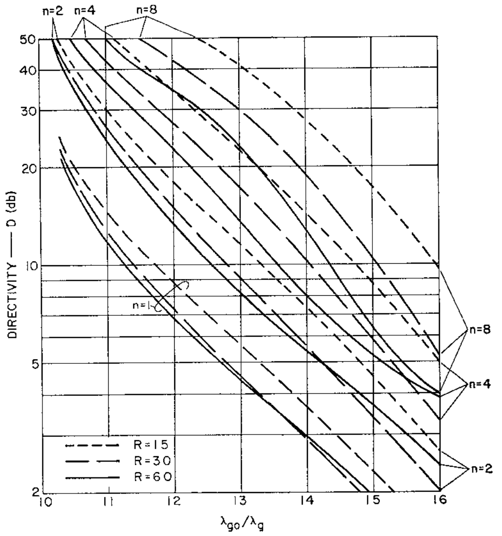

Fig. 15—Directivity characteristics of some maximally flat couplers. (Immittances of these couplers are given in Tables I and II.)

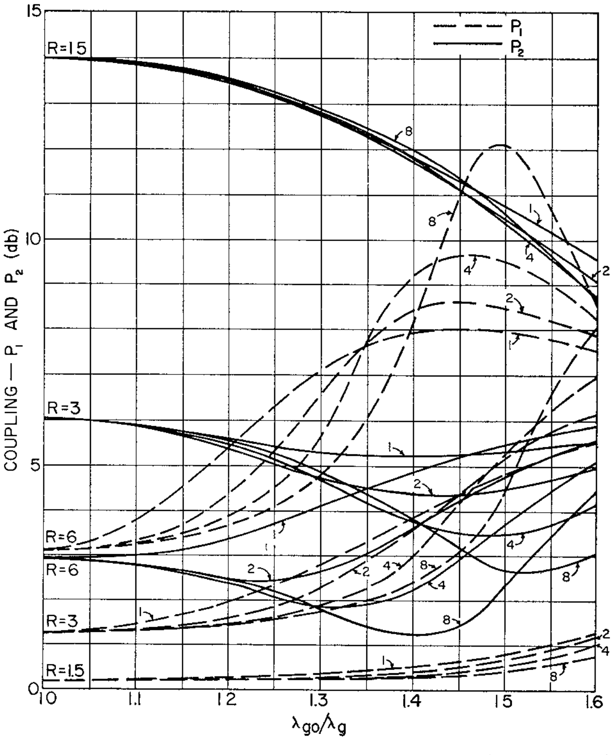

Fig. 16—Coupling characteristics of some maximally flat couplers. (Immittances of these couplers are given in Tables I and II.)

Its analyzed performance is reproduced in Fig. 17, and it is found to conform very closely to the specifications. (Again, this is plotted only for one side of band center, since the response is symmetrical as plotted.) From Fig. 17, the analyzed performance over the 24 per cent bandwidth is: Maximum VSWR, 1.07 (1.08 was predicted); Minimum directivity, 26 db (25 db was predicted); Couplings P_1 and P_2 both within ± 0.2 db of 3 db (± 0.15 db was predicted).

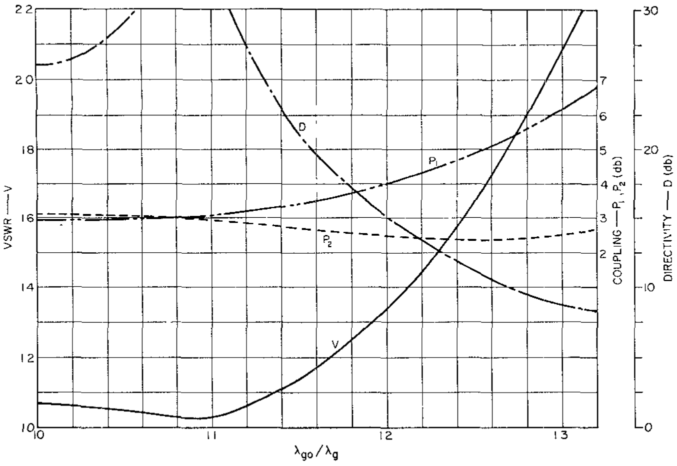

Fig. 17—Computed performance of three-branch ($n=2$) coupler of Example IV-1.

V. Further Numerical Examples

In applying any approximate design procedure, the question inevitably arises: How accurate is it? One way to answer this question is to analyze numerically the performance of some representative cases. A few more of these sample cases which were analyzed are reproduced in this section.

Example V-1—Design of a 6-db Coupler

Optimize a five-branch 6-db coupler over a 25 per cent fractional bandwidth. Estimate the maximum VSWR and the minimum directivity over this band, and the variation in coupling.

For a well-matched coupler, a coupling coefficient of 6 db requires $R=3$, by Table I. The bandwidth contraction factor, β, is 0.64 from Fig. 13. For a coupler fractional bandwidth $w_b=25$ per cent, we therefore require $w_q=0.25/0.64=0.4$. From data on quarter-wave transformers for $n=4$ (Table III of Young[12]) the maximum pass-band VSWR ($V'_{\max}$) for the quarter-wave transformer will be less than 1.01. Therefore $V_{\max}$ will be less than 1.005 for the branch-guide coupler, by (8c), since $P_2=6$ db. The directivity by (9c) should be better than $-20\log_{10}(0.005/2\times 0.866)-6=44$ db, since $P_1=1.25$ db for a well-matched 6-db branch-guide coupler.

This coupler was designed by chart, and then constructed in waveguide according to the impedance values so obtained. The impedances were later recomputed more accurately by programming a digital computer after the coupler had already been built. (These later and more accurate values are shown in brackets below, as they were not used in constructing the coupler.) It was found that

$$\left.\begin{array}{ll} K_0 = 1.0(1.0) & H_1 = 0.070(0.0688) \\ K_1 = 1.036(1.0367) & H_2 = 0.274(0.2823) \\ K_2 = 1.127(1.1323) & H_3 = 0.450(0.4522) \end{array}\right\}. \tag{14}$$

A comparison between the first set of numbers (calculated by chart) and the numbers in brackets (calculated by digital computer) gives an indication of the accuracy obtainable by chart. Here it is about 2 per cent in H and in $(K-1)$ on the average.[21]

The analyzed performance of the coupler with the impedances in brackets (obtained by digital computer) conforms very closely to the predicted values of maximum VSWR and minimum directivity over the 25 per cent pass band: its computed values by analysis were respectively 1.007 for the maximum VSWR (compare 1.005 predicted), and 43 db for the minimum directivity (compare 44 db predicted); the center frequency coupling is exactly 6.02 db. The analyzed performance of the coupler with the first set of impedances (obtained by chart) is little changed though the maximum VSWR in the 25 per cent pass band is now 1.01, and the minimum directivity 34 db; the center frequency coupling is 6.1 db. Junction discontinuity effects and mechanical tolerances may be expected to have a greater effect on performance than inaccuracies due to using the chart.

The analyzed performance of the coupler (designed by chart) is shown in Fig. 18, together with the experimental results in waveguide, which will be described later.

The variation in coupling may be estimated from Fig. 16. It is seen from the curve for $n=4$, $R=3$, that P_2 changes from 6 db at band-center by 0.2 db to 5.8 db at the 25 per cent band-edges ($\lambda_{g0}/\lambda_g=1\pm 0.125$). The analyzed performance (Fig. 18) shows a like change of about 0.2 db from band-center (6.1 db) to band-edges (5.9 db).

Example V-2—Design of a 0-db Coupler

Design a thirteen-branch 0-db coupler making use of the five-branch coupler ($n=4$) of the previous example. What maximum insertion loss would one expect over a 40 per cent fractional bandwidth?

The 0-db coupler can be put together from three 6-db couplers. By merging the end branches of adjacent couplers, two branches can be eliminated, and the 0-db coupler has thus thirteen branches. Its imittances, based on (14), are as follows:

$$\begin{aligned} K_0 &= K_{13} = 1.0 \\ K_1 &= K_4 = K_5 = K_8 = K_9 = K_{12} = 1.036 \\ K_2 &= K_3 = K_6 = K_7 = K_{10} = K_{11} = 1.127 \\ H_1 &= H_{13} = 0.070 \\ H_2 &= H_4 = H_6 = H_8 = H_{10} = H_{12} = 0.274 \\ H_3 &= H_7 = H_{11} = 0.450 \\ H_5 &= H_9 = 0.140 \end{aligned} \tag{15}$$

The coupler of Example V-1 varied in coupling from 6.1 db at band-center to 5.5 db over a 40 per cent fractional bandwidth (λ_{g0}/λ_g from 0.8 to 1.2), as can be seen from Fig. 18. The performance of this coupler is shown in Figs. 19 and 20, together with the experimental results in waveguide, which will be described

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[21] This was one of several early examples worked out quickly on the charts. With a little more care, the accuracy is about 1 per cent.

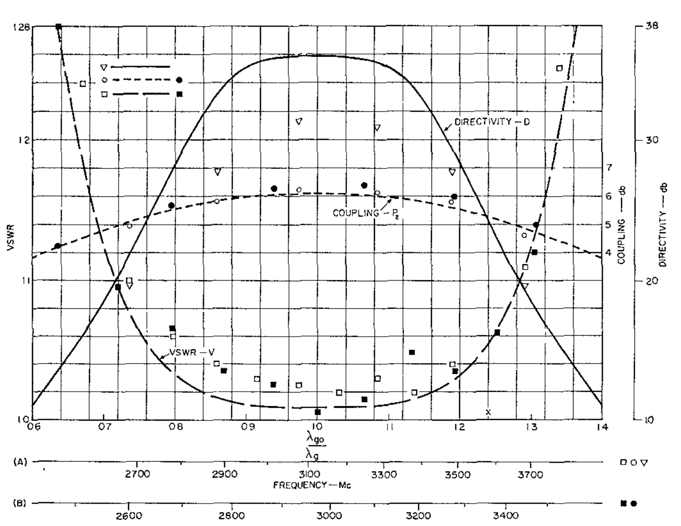

Fig. 18—Performance of five-branch ($n=4$) coupler of Example V-1. (Lines are computed; points are measured on experimental models shown in Figs. 22–24.)

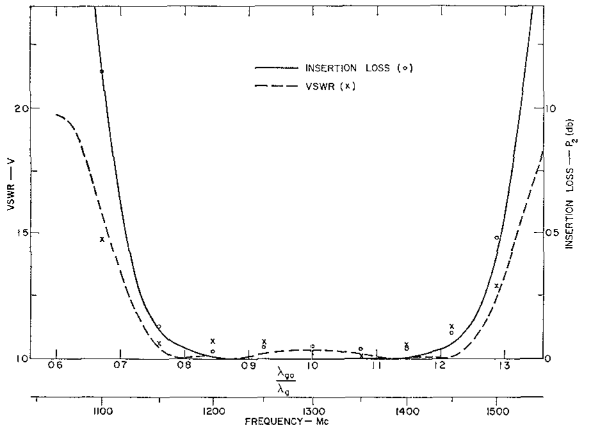

Fig. 19—Insertion loss P_2 (db) and VSWR of 0-db coupler of Example V-2. (Lines are computed; points are measured on experimental model shown in Figs. 24 and 25.)

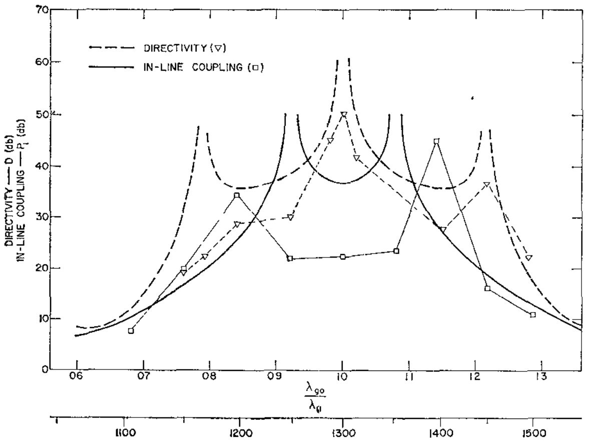

Fig. 20—Directivity D and in-line coupling P_1 of 0-db coupler of Example V-2. (Lines are computed; points are measured on experimental model shown in Figs. 24 and 25.)

later. It is seen that the insertion loss upon analysis is indeed better than 0.05 db over a 40 per cent fractional bandwidth. (The experimentally measured points also agree very closely.)

VI. Experimental Results

A. The 6-db Coupler

1) *Construction:* The 6-db coupler of Example V-1 was constructed in S band. This coupler has five branches. Since waveguide T-junctions are series junctions, the immittances K_i and H_i are impedances; however, since waveguide T-junctions are not perfect series junctions, they can be represented by an equivalent circuit.[22] The details of the calculation[23] are straightforward, but tedious, and are not reproduced here.

In order to obtain straight top and bottom walls as shown in Fig. 1, and yet maintain the correct impedances K_i, branch lengths, some of which differed slightly from the lengths calculated theoretically, have to be accepted.

The dimensions so calculated are shown in Fig. 21. The coupler was constructed in two halves as shown in Fig. 22. Three jig-plates of aluminum were used to make a U-shaped channel, in which six aluminum blocks were placed and bolted down to form the waveguide channels. The end blocks contain the transformers from the waveguide height of 1.420 inches to 1.340 inches. The depth of all the channels is half of 2.840 inches, or 1.420 inches. The two pieces shown in Fig. 22 were finally superimposed and bolted together to form the 6-db coupler.

The measured performance of the completed coupler is shown by the light points in Fig. 18, which go with the frequency scale (A) near the bottom of Fig. 18. Plotted on a (λ_{g0}/λ_g) scale, the points fit the computed curves very closely; however, the center frequency is 3125 Mc instead of the design value of 2975 Mc. This discrepancy is thought to result from the relatively large b-dimensions which, for instance, make the length of an outline edge on the two center squares in Fig.21 only about one-seventh wavelength. Thus higher-order modes could be set up, giving rise to interaction effects at such close spacings. If this explanation is correct, then lower waveguide heights (smaller b-dimensions) would result in better design accuracy. However, this was not attempted since the coupler was to be used at high powers where large waveguide heights are an advantage.

All branch lengths and spacings, nominally one-quarter wavelength, were then scaled in the ratio of the guide wavelengths to reduce the center frequency from 3125 to 2975 Mc, and the coupler was tested again. Its center frequency moved down as expected, but the

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[22] N. Marcuvitz, "Waveguide Handbook," M.I.T. Rad. Lab. Ser., vol. 10, McGraw-Hill Book Co., Inc., New York, N. Y., pp. 336–350; 1951. (See especially Fig. 6.1-2, p. 338.)

[23] L. Young, "The Design of Branch Guide Directional Couplers for Low and High Power Applications," Stanford Research Inst., Menlo Park, Calif., Tech. Note 3, SRI Project 3478, Contract AF 30(602)-2392; February, 1962.

coupling there became stronger, going from 6.1 to 5.8 db. Since the coupling becomes stronger still at off-center frequencies, it was decided to reduce the branch heights to weaken the coupling by 0.5 db at center frequency, changing the 5.8 db to 6.3 db coupling. The new dimensions calculated are shown in Fig. 23. The measured results are shown by the black points in Fig. 18, which go with the frequency scale (B) at the bottom of Fig. 18. It is seen that this coupler gives the desired center frequency and its performance closely follows the computed curves.

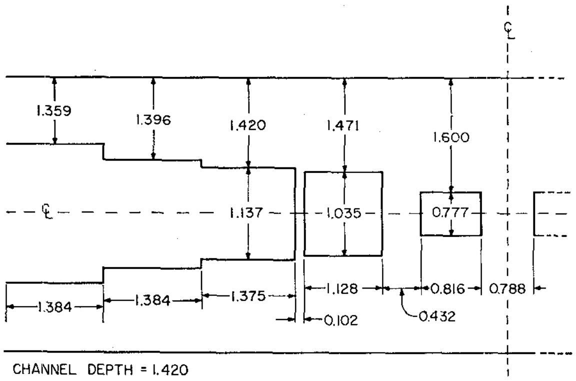


Fig. 21—Dimensions of first S-band 6-db coupler—based on Example V-1.

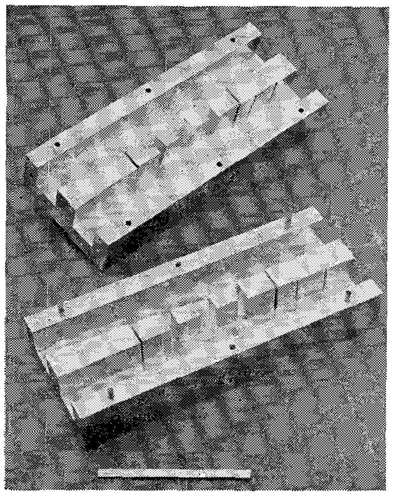
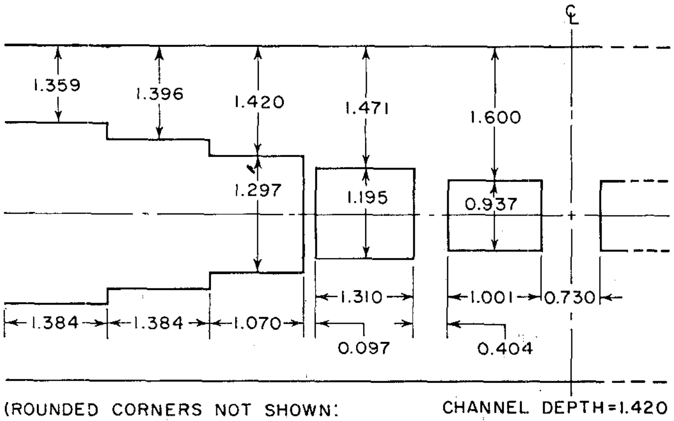


Fig. 22—Exploded view of 6-db experimental coupler.



(ROUNDED CORNERS NOT SHOWN: SEE TEXT) CHANNEL DEPTH=1.420

Fig. 23—Dimensions of S-band$_{6\text{-db}}$ coupler after modifications.

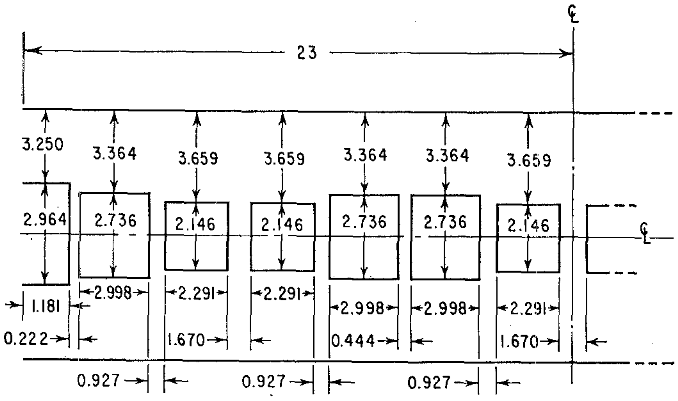
All edges orthogonal to the electric field of the TE_{10} mode were then rounded off to increase the power handling capacity. The edges of the eight rectangular blocks were rounded to a radius of $\frac{1}{8}$ inch; the edges of the four outside blocks, on the faces defining the outside edges of the narrowest branches, were rounded to a radius of $\frac{1}{16}$ inch. The rounding did not introduce any measurable change in the coupler characteristics (Fig. 18).

B. The 0-db Coupler

1. Construction: The 0-db coupler is constructed following the procedure of Example V-2. It is based on the 6-db experimental coupler just described, but scaled from S band to L band, with the ratio of the two guide wavelengths at center frequency as the scaling factor. The ratio of the guide wavelengths was made equal to the ratio of the a dimensions of WR-650 and WR-284.

The coupler was constructed of aluminum jig-plate in the fashion described for the 6-db coupler. The overall length was close to 4 feet, so that it was made in two flanged sections, which bolted together. The dimensions of one-half the coupler are given in Fig. 24. A photograph of the entire coupler is shown in Fig. 25.

The measured performance of this coupler is shown in Figs. 19 and 20, and it is found to agree very closely with the computed curves.



(ROUNDED CORNERS NOT SHOWN: SEE TEXT) CHANNEL DEPTH=3.250

Fig. 24—Dimensions of 0-db coupler.

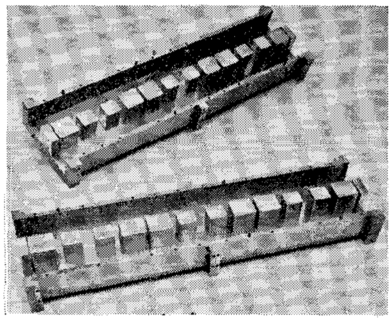


Fig. 25—Exploded view of first 0-db experimental coupler.

2. Power-Handling Capacity: The edges perpendicular to the electric field were rounded to a $\frac{1}{4}$-inch radius, except for an $\frac{1}{8}$-inch radius on the end blocks, to increase the power-handling capacity. This was found to have no measurable effect on the performance.

High-power tests were made using the Eimac X832 experimental Klystron amplifier, which operates at L-band and supplies 2-μsec pulses at 60 pps. A cobalt-60 radioactive source was placed under the coupler to insure that the air was ionized at points most susceptible to breakdown. The branch-guide coupler was filled with air at atmospheric pressure. It did not break down at the peak power available, which was 5.1 Mw. The VSWR of the water-load at the output of the coupler was 1.3, but as no phase shifter was available, it is not known how much it helped or hurt the power handling capacity. However, it is probably safe to say that this coupler would handle at least 5 Mw of peak power in air at atmospheric pressure with a matched termination.

3. Suppression of Spurious Frequencies: The use of 0-db and 3-db branch guide couplers as harmonic pads for the suppression of spurious frequencies is described in Young,[24] and more fully in Young.[23]

VIII. Conclusion

A design method for branch-guide couplers has been developed which gives close to optimum performance over a given pass band. The method was tested by analyzing the performance of several numerical designs, then constructing two couplers in waveguide and comparing the actual with the computed characteristics. The agreement was close. The design procedure is facilitated by a new chart constructed for this purpose; alternatively, maximally flat cases can be worked out from the tables in the Appendix by interpolation. The L-band 0-db coupler has passed over 5 Mw of peak power in air at atmospheric pressure without any sign of arcing.

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Appendix

Tables of Maximally Flat Branch-Guide Coupler Immittances

The notation for the immittances H_i and K_i is shown in Fig. 2. For a coupler with series T-junctions, H_i and K_i are both characteristic impedances (*e.g.*, for E-plane junctions in waveguide); for a coupler with shunt T-junctions, H_i and K_i are both characteristic admittances (*e.g.*, for coaxial or strip transmission lines).

TABLE I

Branch-Guide Coupler Immittances for $n=1$ Section (Two Branches)

$n=1$		
R	K_1	H_0
1.25	1.006	0.1119
1.50	1.021	0.2040
2.00	1.061	0.3535
2.50	1.108	0.4732
3.00	1.155	0.5775
4.00	1.250	0.7500
5.00	1.341	0.894
6.00	1.429	1.021
8.00	1.592	1.238
10.00	1.739	1.423

TABLE II

Immittances of Maximally Flat Branch-Guide Couplers for $n=2$ to 8 Sections (Three to Nine Branches)

R	K_1	K_2	K_3	K_4	H_1	H_2	H_3	H_4	H_5
$n=2$									
1.50	1.0153				0.1010	0.2062			
2.00	1.0449				0.1715	0.3639			
2.50	1.0783				0.2251	0.4983			
3.00	1.1124				0.2679	0.6188			
4.00	1.1785				0.3333	0.8333			
5.00	1.2399				0.3819	1.0249			

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[24] L. Young, "The application of branch guide couplers to the suppression of spurious frequencies," Proc. 4th Natl. Symp. on Radio Frequency Interference, San Francisco, Calif., June 28–29, 1962.

TABLE II (*Cont'd*)

R	K_1	K_2	K_3	K_4	H_1	H_2	H_3	H_4	H_5
$n=2$ (*cont'd*)									
6.00	1.2965				0.4202	1.2008			
8.00	1.3978				0.4775	1.5196			
10.00	1.4861				0.5194	1.8070			
$n=3$									
1.50	1.0089	1.0206			0.0501	0.1539			
2.00	1.0258	1.0606			0.0840	0.2694			
2.50	1.0446	1.1067			0.1086	0.3656			
3.00	1.0634	1.1546			0.1274	0.4498			
4.00	1.0988	1.2499			0.1542	0.5957			
5.00	1.1307	1.3416			0.1732	0.7220			
6.00	1.1594	1.4288			0.1854	0.8351			
8.00	1.2087	1.5909			0.2029	1.0344			
10.00	1.2501	1.7392			0.2138	1.2091			
$n=4$									
1.50	1.0047	1.0176			0.0249	0.1017	0.1548		
2.00	1.0137	1.0517			0.0415	0.1750	0.2738		
2.50	1.0235	1.0907			0.0533	0.2331	0.3758		
3.00	1.0333	1.1307			0.0620	0.2814	0.4676		
4.00	1.0514	1.2093			0.0740	0.3596	0.6326		
5.00	1.0675	1.2835			0.0817	0.4220	0.7811		
6.00	1.0816	1.3530			0.0870	0.4743	0.9185		
8.00	1.1057	1.4795			0.0935	0.5589	1.1698		
10.00	1.1256	1.5924			0.0971	0.6265	1.3986		
$n=5$									
1.50	1.0024	1.0124	1.0206		0.0124	0.0630	0.1285		
2.00	1.0070	1.0363	1.0606		0.0206	0.1069	0.2259		
2.50	1.0121	1.0630	1.1067		0.0264	0.1400	0.3079		
3.00	1.0170	1.0902	1.1546		0.0306	0.1662	0.3804		
4.00	1.0262	1.1421	1.2499		0.0363	0.2058	0.5078		
5.00	1.0342	1.1899	1.3416		0.0398	0.2348	0.6196		
6.00	1.0413	1.2335	1.4288		0.0422	0.2573	0.7210		
8.00	1.0532	1.3106	1.5909		0.0450	0.2904	0.9019		
10.00	1.0630	1.3769	1.7392		0.0464	0.3140	1.0625		
$n=6$									
1.50	1.0012	1.0079	1.0185		0.0062	0.0376	0.0957	0.1290	
2.00	1.0035	1.0230	1.0544		0.0103	0.0631	0.1658	0.2284	
2.50	1.0061	1.0397	1.0956		0.0131	0.0817	0.2225	0.3138	
3.00	1.0086	1.0564	1.1380		0.0152	0.0959	0.2707	0.3908	
4.00	1.0132	1.0878	1.2216		0.0180	0.1162	0.3509	0.5295	
5.00	1.0172	1.1161	1.3010		0.0197	0.1301	0.4170	0.6549	
6.00	1.0208	1.1414	1.3758		0.0208	0.1402	0.4738	0.7712	
8.00	1.0268	1.1852	1.5128		0.0221	0.1539	0.5689	0.9846	
10.00	1.0317	1.2220	1.6361		0.0228	0.1626	0.6477	1.1796	
$n=7$									
1.50	1.0006	1.0047	1.0143	1.0206	0.0031	0.0218	0.0665	0.1126	
2.00	1.0018	1.0137	1.0419	1.0606	0.0051	0.0364	0.1135	0.1983	
2.50	1.0030	1.0236	1.0731	1.1067	0.0065	0.0468	0.1499	0.2709	
3.00	1.0043	1.0334	1.1048	1.1546	0.0076	0.0546	0.1795	0.3355	
4.00	1.0066	1.0516	1.1662	1.2499	0.0089	0.0653	0.2258	0.4497	
5.00	1.0086	1.0678	1.2233	1.3416	0.0098	0.0724	0.2612	0.5509	
6.00	1.0104	1.0822	1.2759	1.4288	0.0103	0.0772	0.2898	0.6431	
8.00	1.0134	1.1067	1.3699	1.5909	0.0110	0.0834	0.3339	0.8090	
10.00	1.0159	1.1270	1.4520	1.7392	0.0113	0.0869	0.3672	0.9574	
$n=8$									
1.50	1.0003	1.0027	1.0101	1.0190	0.0015	0.0124	0.0440	0.0895	0.1129
2.00	1.0009	1.0079	1.0293	1.0559	0.0025	0.0207	0.0743	0.1558	0.2000
2.50	1.0015	1.0135	1.0508	1.0982	0.0032	0.0265	0.0969	0.2100	0.2748
3.00	1.0021	1.0191	1.0724	1.1419	0.0037	0.0308	0.1146	0.2567	0.3425
4.00	1.0033	1.0295	1.1134	1.2282	0.0044	0.0366	0.1408	0.3357	0.4644
5.00	1.0043	1.0386	1.1508	1.3104	0.0049	0.0402	0.1595	0.4021	0.5749
6.00	1.0052	1.0467	1.1846	1.3881	0.0051	0.0427	0.1737	0.4601	0.6775
8.00	1.0067	1.0603	1.2436	1.5308	0.0055	0.0457	0.1939	0.5591	0.8661
10.00	1.0080	1.0715	1.2940	1.6598	0.0056	0.0474	0.2078	0.6426	1.0388